S.S.L.C.
Mathematics

SETS

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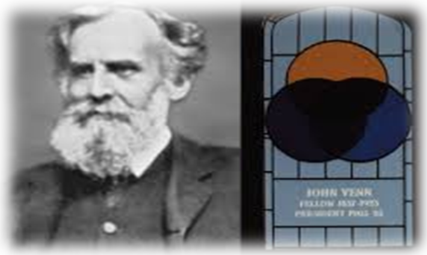
Chapter – 1 Sets

Main Points to be remember



JOHN VENN

1. Union of sets
- 1. Commutative property [$A \cup B = B \cup A$]
- 2. Associative property [$(A \cup B) \cup C = A \cup (B \cup C)$]

2. Intersection of sets
- 1. Commutative property [$A \cap B = B \cap A$]
- 2. Associative property [$(A \cap B) \cap C = A \cap (B \cap C)$]

3. Distributive property
- 1. $A \cup (B \cap C) = (A\ B) \cap (A \cup C)$
- 2. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

4. De Morgan's Law
- 1. $(A \cup B)^1 = A^1 \cap B^1$
- 2. $(A \cap B)^1 = A^1 \cup B^1$

5. Cardinality of sets
- 1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- 2. $n(A \cup B) = n(A) + n(B)$

Exercise 2.1

1. Verify the commutative property of union and intersection of sets for the following
   $A = \{l, m\ n, o, p, q\}$, $B = \{m, n, o, r, s, t\}$

Ans: $A \cup B = \{l, m, n, o, p, q\} \cup \{m, n, o, r, s, t\}$
$A \cup B = \{l, m, n, o, p, q, r, s, t\}$ - - - - - - - - - - - (1)
$B \cup A = \{m, n, o, r, s, t\} \cup \{l, m\ n, o, p, q\}$
$A \cup B = \{l, m, n, o, p, q, r, s, t\}$ - - - - - - - - - - - (2)
From (1) and (2) $A \cup B = B \cup A$
∴ Union of sets is commutative.
$A \cap B = \{l, m, n, o, p, q\} \cap \{m, n, o, r, s, t\}$
$A \cap B = \{m, n, o,\}$ - - - - - - - - - - - (1)
$B \cap A = \{m, n, o, r, s, t\} \cap \{l, m\ n, o, p, q\}$
$B \cap A = \{m, n, o,\}$ - - - - - - - - - - - (2)

From (1) and (2) $A \cap B = B \cap A$

$\therefore$ Intersection of sets is commutative

2. Given P = {a, b, c, d, e} Q = {a, e, i, o, u} and R = {a, c, e, g}, Verify associative property of union and intersection of sets.

Ans: (PUQ)UR = [{ a, b, c, d, e }U{ a, e, i, o, u }]U{ a, c, e, g }

(PUQ)UR = {a, b, c, d, e, i, o, u} U {a, c, e, g}

(PUQ)UR = {a, b, c, d, e, g, i, o, u} - - - - - - - - - - - - - (1)

PU(QUR)= { a, b, c, d, e }U[{ a, e, i, o, u }U{ a, c, e, g }]

PU (QUR) = {a, b, c, d, e} U {a, c, e, g, i, o, u}

(PUQ)UR = {a, b, c, d, e, g, i, o, u} - - - - - - - - - - - - - (2)

From (1) and (2)

(PUQ)UR = PU (QUR)

$\therefore$ The union of sets is associative.

$(P \cap Q) \cap R$ = [{a, b, c, d, e} ∩ {a, e, i, o, u}] ∩ {a, c, e, g}

$(P \cap Q) \cap R$ = {a, e} ∩ {a, c, e, g}

$(P \cap Q) \cap R$ = {a, e} - - - - - - - - - - - - - (1)

$P \cap (Q \cap R)$ = {a, b, c, d, e} ∩ [{a, e, i, o, u} ∩ {a, c, e, g}]

$P \cap (Q \cap R)$ = {a, b, c, d, e} ∩ {a, e}

$P \cap (Q \cap R)$ = {a, e} - - - - - - - - - - - - - (2)

From (1) and (2) AUB = BUA

$(P \cap Q) \cap R = P \cap (Q \cap R)$

$\therefore$ The intersection of sets is.

3. If A = { -3, -1, 0, 4, 6, 8, 10 }, B = { -1, -2, 3, 4, 5, 6 } and C = { -6, -4, -2, 2, 4, 6 }, Show that AU(B∩C) = (AUB)∩(AUC) .

Ans:

LHS: AU (B∩C) = {-3, -1, 0, 4, 6, 8, 10} U [{-1, -2, 3, 4, 5, 6} ∩ {-6, -4, -2, 2, 4, 6}]

AU (B∩C) = {-3, -1, 0, 4, 6, 8, 10} U {-2, 4, 6}

AU (B∩C) = {-3, -2, -1, 0, 4, 6, 8, 10} - - - - - - - - - - - - - (1)

RHS: (AUB)∩(AUC) = [{-3,-1,0,4,6,8,10 }U{-1,-2,3,4,5,6}]∩[{-3,-1,0,4,6,8,10}U{-6,-4,-2,2,4,6}]

(AUB)∩(AUC) = {-3,-2,-1,0,3,4,5,6,8,10 }∩{-6,-4,-3,-2,-1,0,2,4,6,8,10}

(AUB)∩ (AUC) = {-3, -2, -1, 0, 4, 6, 8, 10} - - - - - - - - - - - - (2)

From (1) and (2) AU (B∩C) = (AUB) ∩ (AUC)

4. If U = { 4,8, 12, 16, 20, 24, 28 }, A = { 8, 16, 24 }, B = { 4, 16, 20, 28 } Verify that (i). $(AUB)^1 = A^1 \cap B^1$ (ii) $(A \cap B)^1 = A^1 U B^1$

Ans: (i).

LHS: $(AUB)^1$ = [{8, 16, 24} U {4, 16, 20, 28}]^1

$(AUB)^1$ = [{4, 8, 16, 20, 24, 28}]^1

$(AUB)^1$ = {12} - - - - - - -- - - - - - - - - (1)

RHS: $A^1 \cap B^1$ = [{8, 16, 24}]^1 ∩ [{4, 16, 20, 28}]^1

$A^1 \cap B^1$ = {4, 12, 20, 28} ∩ {8, 12, 24,}

$A^1 \cap B^1$ = {12} - - - - - - - -- - - - - - - - (2)

From (1) and (2) $(AUB)^1 = A^1 \cap B^1$

Ans :(ii).

LHS :$(A \cap B)^1$ = [{8, 16, 24} ∩ {4, 16, 20, 28}]^1

$(A \cap B)^1$ = [{16}]^1

$(A \cap B)^1$ = {4, 8, 12, 20, 24, and 28} - ---- ---------- (1)

RHS:$A^1 U B^1$ = [{8, 16, 24}]^1 U [{4, 16, 20, 28}]^1

$A^1 U B^1$ = {4, 12, 20, 28} U {8, 12, 24}

$A^1 U B^1$ = {4, 8, 12, 20, 24, 28} ---------------- (2)

From (1) and (2) $(A \cap B)^1 = A^1 U B^1$

5. If A = { 1, 2, 3 }, B = { 2, 3, 4, 5 }, C = { 2, 4, 5, 6 } are the subset of U = { 1, 2, 3, 4, 5, 6, 7, 8 }, Verify De Morgan's Law.1. $(AUB)^1 = A^1 \cap B^1$ 2.$(A \cap B)^1 = A^1 U B^1$

Ans :(1).

LHS: $(AUB)^1 = [\{1, 2, 3\} \cup \{2, 3, 4, 5\}]^1$

$(AUB)^1 = [\{1, 2, 3, 4, 5\}]^1$

$(AUB)^1 = \{6, 7, 8\}$ - - - - - - - -- - - - - - - - - (1)

RHS: $A^1 \cap B^1 = [\{1, 2, 3\}]^1 \cap [\{2, 3, 4, 5\}]^1$

$A^1 \cap B^1 = \{4, 5, 6, 7, 8\} \cap \{1, 6, 7, 8\}$

$A^1 \cap B^1 = \{6, 7, 8\}$ - - - - - - - -- - - - - - - - - (2)

From (1) and (2) $(AUB)^1 = A^1 \cap B^1$

Ans :(ii).

LHS: $(A \cap B)^1 = [\{1, 2, 3\} \cap \{2, 3, 4, 5\}]^1$

$(A \cap B)^1 = [\{2, 3\}]^1$

$(A \cap B)^1 = \{1, 4, 5, 6, 7, \text{ and } 8\}$ - ---- ---------- (1)

RHS: $A^1 U B^1 = [\{1, 2, 3\}]^1 \cup [\{2, 3, 4, 5\}]^1$

$A^1 U B^1 = \{4, 5, 6, 7, 8\} \cup \{1, 6, 7, 8\}$

$A^1 U B^1 = \{1, 4, 5, 6, 7, 8\}$ ----------------- (2)

From (1) and (2) $(A \cap B)^1 = A^1 U B^1$

6. If A = { 2, 3, 5, 7, 11, 13 } and B = { 5, 7, 9, 11, 15 } are the subsets of U = { 2, 3, 5, 7, 9, 11, 13, 15 }, Verify De Morgan's Law

Ans :(1). $(AUB)^1 = A^1 \cap B^1$

LHS: $(AUB)^1 = [\{2, 3, 5, 7, 11, 13\} \cup \{5, 7, 9, 11, 15\}]^1$

$(AUB)^1 = [\{2, 3, 5, 7, 9, 11, 13, 15\}]^1$

$(AUB)^1 = \{\quad\}$ - - - - - - - - -- - - - - - - - - (1)

RHS: $A1 \cap B^1 = [\{2, 3, 5, 7, 11, 13\}]^1 \cap [\{5, 7, 9, 11, 15\}]^1$

$A^1 \cap B^1 = \{9, 15\} \cap \{3, 13\}$

$A^1 \cap B^1 = \{\quad\}$ - - - - - - - - -- - - - - - - - - (2)

From (1) and (2) $(AUB)^1 = A^1 \cap B^1$

Ans :(ii). $(A \cap B)^1 = A^1 U B^1$

LHS: $(A \cap B)^1 = [\{2, 3, 5, 7, 11, 13\} \cap \{5, 7, 9, 11, 15\}]^1$

$(A \cap B)^1 = [\{5, 7, 11\}]^1$

$(A \cap B)^1 = \{2, 3, 9, 13, \text{ and } 15\}$ - ---- ---------- (1)

RHS: $A^1 U B^1 = [\{2, 3, 5, 7, 11, 13\}]^1 \cup [\{5, 7, 9, 11, 15\}]^1$

$A^1 U B^1 = \{9, 15,\} \cup \{2, 3, 13\}$

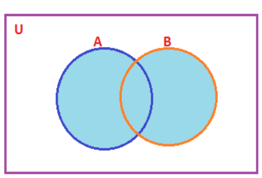
$A^1 U B^1 = \{2, 3, 9, 13, 15\}$ ----------------- (2)

From (1) and (2) $(A \cap B)^1 = A^1 U B^1$

7. Draw Venn diagrams to illustrate the following:

i. (AUB) ii. $(AUB)^1$ iii. $A^1 \cap B$ IV. $A \cap B^1$ v. A\B VI. A∩ (B\C) vii. AU(B∩C) viii. C∩ (BUA) ix. C∩ (B\A) x. A\ (B∩C) xi. (A\B) U (A\C) xii. (AUB)\ (AUC)

i.



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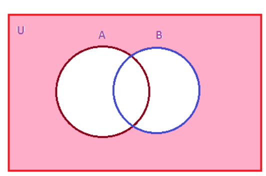

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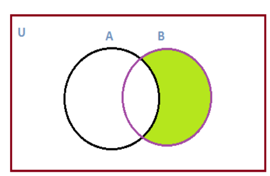

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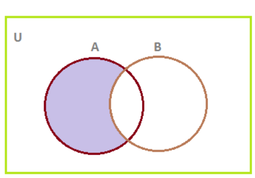

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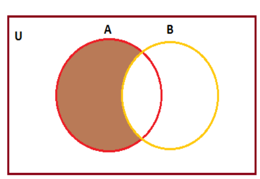
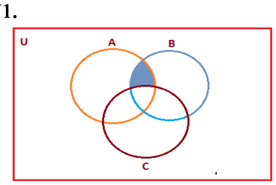


vi.



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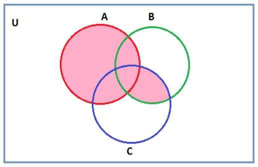

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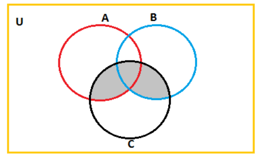

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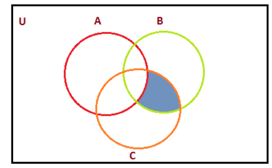
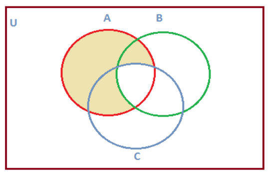
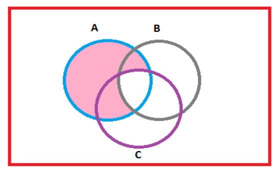


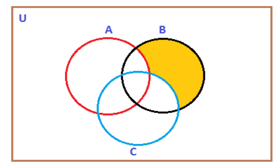
x.



xi.



xii.



ILLUSTRATIVE EXAMPLES

Example 1: If A and B are two sets such that n (A) = 27, n (B) = 35 and n (AUB) = 50, find n (A ∩B).

Sol: n (A) = 27, n (B) = 35, n (A UB) = 50, n (A ∩B) =?

n (AUB) = n (A) + n (B) – n (A ∩B)

50 = 27 + 35 – n (A ∩B)

n (A ∩B) = 62 – 50

∴ n (A ∩B) = 12

Example 2 : In a group of students, 75 scored first class marks in Kannada, 70 scored first class marks in Social science and 45 scored first class in both the subjects. Find the number of students in the group.

Sol: Number of students who scored I class in Kannada, n(K)= 75

Number of students who scored I class in Social science n(S) = 70

Number of students who scored I class in both the subjects n(K □□S) = 45

Number of students = n(KUS) = ?

n(KUS) = n(K) + n(S) – n(K∩S)

∴n(KUS) = 75 + 70 – 45 = 145 – 45 = 100.

So, there are 100 students in the group.

Example 3: Among 520 members in a village, 360 members were engaged in cattle rearing and 280 members with poultry farming and 180 were doing both. How many people are (i) not involved in either of the work ? (ii) engaged only in poultry farming.

Sol: Cattle rearing, n (A) = 360; Poultry farming (B) = 280; both n (A∩B) = 180

We know, n (AUB) = n (A) + n (B) – n (A∩B)

n (AUB) = 360 + 280 - 180

n (AUB) = 360 + 100 = 460

∴Nnumber of people who are engaged in the work is 460.

(I) ∴Number of people who are not engaged in either of the work is = 520 – 460 = 60.

∴60 members of the village are not involved in either of the work.

(ii) Number of people engaged only in poultry farming = n (B) \ n (A∩B)

= 280 – 180 = 100

Exercise 2.2

I. Solve the following problems and verify the data in each case by drawing Venn diagrams.

1. If A and B are the sets such that $n(A) = 37$, $n(B) = 26$ and $n(A \cup B) = 51$, find $n(A \cap B)$

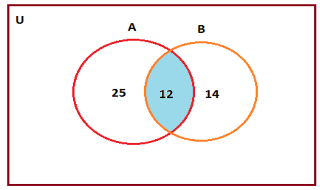
Ans: $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$n(A \cap B) = 37 + 26 - 51$

$n(A \cap B) = 63 - 51$

$n(A \cap B) = 12$



2. In a group of 50 persons, 30 like tea, 25 like coffee and 16 like both. How many like (i) Either tea or coffee (ii) neither tea nor coffee

Ans: A – {like tea}, B – {like coffee}

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$N(A \cup B)$ = {No.of persons like either tea or coffee}, $n(A)$ =No.of persons like tea,

$N(B)$ =No.of persons like coffee. $N(A \cap B)$ = No.of persons like both tea and coffee.

Ans :(i) $n(A \cup B) = 30 + 25 - 16$

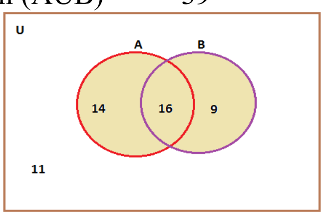
$N(A \cup B) = 55 - 16$

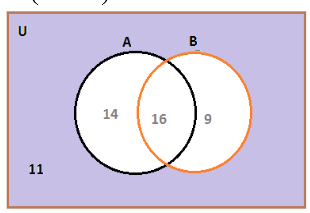
$n(A \cup B) = 39$

Ans :(ii) $n(A \cup B)^1 = 50 - n(A \cup B)$

$n(A \cup B)^1 = 50 - 39$

$n(A \cup B)^1 = 11$





3. In a group of passengers, 100 know Kannada, 50 know English and 25 know both. If passengers know either Kannada or English. How many passengers are in the group?

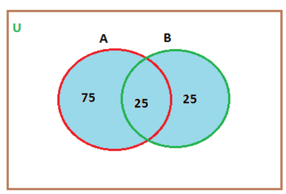
Ans: K – {Passengers know Kannada}, E – {Passengers know English}

$n(K) = 100$, $n(E) = 50$, $n(K \cap E) = 25$

$n(K \cup E) = n(K) + n(E) - n(K \cap E)$

$n(K \cup E) = 100 + 50 - 25$

$n(K \cup E) = 125$



4. In a class, 50 students offered Mathematics, 42 offered Biology and 24 offered both the subjects. Find the number of students, who offer i) Mathematics only (ii) Biology only. Also find the number of students in the class.

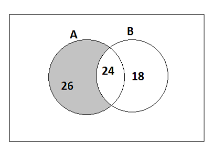
Ans: M = {students offered Mathematics}, B = {students offered Biology}

$n(M) = 50$, $n(B) = 42$, $n(M \cap B) = 24$

(i).Number of students offer Mathematics only

$= n(M) - n(M \cap B)$

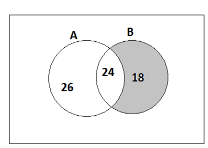
$= 50 - 24 = 26$



(ii). Number of students offer Biology only

$= n(B) - n(M \cap B)$

$= 42 - 24 = 18$



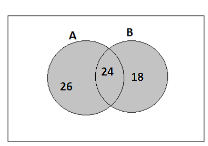
(iii). Total number of students:

$n(M \cup B) = n(M) + n(B) - n(M \cap B)$

$= 50 + 42 - 24$

$= 92 - 24$

$= 68$



5. In a medical examination of 150 people, it was found that 90 had eye problem, 50 had heart problem and 30 had both complaints. What percentage of people had either eye problem or heart problem?

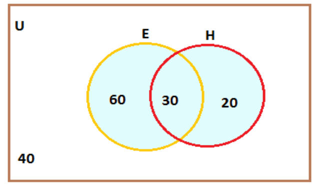
Ans: E = {People with eye problem}, H = {People with heart problem}

n (E) = 90, n (H) =50, n (E∩H) =30

n (E) + n (H) - n (E∩H)

90 + 50 - 30

110 ⇒ 73.33%



II. Solve by drawing Venn diagrams only.

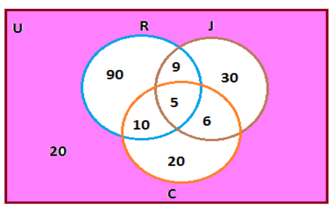
1. A radio station surveyed 190 audience to determine the types of music they liked. The survey revealed that 114 liked rock music, 50 liked folk music, 41 liked classical music, 14 liked rock music and folk music, 15 liked rock music and classical music, and 11 liked classical music and folk music. 5 liked all the three types of music. Find
   (i) How many did not like any of the 3 types?
   (ii) How many liked any two types only?
   (iii) How many liked folk music but not rock music?

Ans: (I). Audience did not like any of the 3 types

R = {Audience liked rock music}, J = {Audience liked folk music},

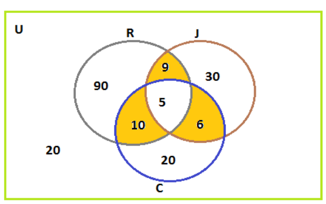
C = {Audience liked classical music}

n(R) = 114, n (J) = 50, n(C) = 41, n (R∩J) = 14, n (J∩C) = 11, n (R∩C) = 15, n (R∩J∩C) = 5



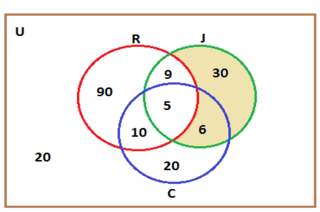
= 20

(ii). Audience liked any two types only



= 25

(iii).Audience liked folk music but not rock music?



= 36

2. In a village, out of 120 farmers, 93 farmers have grown vegetables, 63 have grown flowers, 45 have grown sugarcane, 45 farmers have grown vegetables and flowers, 24 farmers have grown flowers and sugarcane, 27 farmers have grown vegetables and sugarcane. Find how many farmers have grown vegetables, flowers and sugarcane.

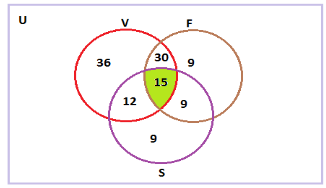
Ans: V = {farmers have grown vegetables}, F = {farmers have grown flowers}, S = {farmers have grown sugarcane}

n (V) = 93, n (F) = 63, n(S) = 45, n (V∩F) = 45, n (F∩S) = 24, n (V∩S) =27

n (VUFUS) = n (V) + n (F) + n(S) – [n (V∩F) + n (F∩S) + n (V∩S)] + n (V∩F∩S)

120 = 93 + 63 + 45 – [45 + 24 + 27] + n (V∩F∩S)

120 = 93 + 63 + 45 – [45 + 24 + 27] + n (V∩F∩S)



120 = 201 – 96 + n (V∩F∩S)

120 = 105 + n (V∩F∩S)

N (V∩F∩S) = 120 – 105

n (V∩F∩S) = 15